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Construction of Mortality Indices

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Abstract

In this article, we suggest that the time-varying parameters in the CBD model are suitable for constructing mortality indices. We note that these indices can be jointly modelled with multivariate time series models. This article forms part of our working paper entitled "The CBD mortality indexes: modeling and applications".

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1. Research background

Similar to stock indices, mortality indices can be constructed to summarise the levels of mortality over time. They have many potential applications. For example, their trends reflect the pace at which the mortality of a population changes. The amount of longevity risk can be estimated from the deviations between the expected and actual paths of a mortality index. One can also use mortality indices to construct standardised mortality-linked securities, such as longevity bonds and swaps. Compared to customised instruments, standardised securities are easier for investors to understand and for the market to develop liquidity.

A number of investment banks have introduced mortality indices in recent years, such as a longevity index by Credit Suisse, the QxX Index by Goldman Sachs, the LifeMetrics Index by JP Morgan, and the Xpect Cohort Index by Deutsche Börse. All these mortality indices are model-free, which may be regarded as an advantage. But without a model, an index can convey only a limited amount of information. For instance, an index based on life expectancy summarises longevity improvement at the aggregate level, while showing nothing on how the underlying mortality curve changes over time. As mortality curve movements are generally not uniform, an annuity provider or a pension plan sponsor may find it hard to structure a longevity hedge with this kind of index. To express mortality curve movements in a non-parametric fashion, a rather large number of indices are needed, e.g. indices linked to mortality rates at specific ages. The use of a large number of mortality indices, however, makes tracking and also developing liquidity difficult.

A model-based method may enhance the information content of mortality indices. Many extrapolative stochastic mortality models have recently been proposed in the literature. When they are applied to historical data, one or more series of time-varying parameters are computed. Mortality forecasts can be made by projecting the parameters into the future. These parameters contain much information and can possibly be taken as mortality indices. But not all models are suitable for building indices. There are three important criteria: (1) the indices should reflect the age patterns of mortality changes; (2) the model should have the 'new-data-invariant' property; (3) the indices should be interpretable. As discussed below, we find that the CBD model appears to be suitable for constructing mortality indices. We also propose to model the resulting mortality indices with the vector autoregressive integrated moving average (VARIMA) models, which take serial- and cross-correlations into account.

2. CBD mortality indices

We use the following notation: $q_{x,t}$ is the probability that a person aged x at time t will die within the next year; $k_t^{(1)}$ and $k_t^{(2)}$ are time-varying parameters; $\bar{x}$ is the mean age over the sample age range. The CBD model [1] is specified as

$$\ln \frac{q_{x,t}}{1-q_{x,t}} = \kappa_t^{(1)} + \kappa_t^{(2)}\left(x - \bar{x}\right).$$

We note that this model has the new-data-invariant property – when new data becomes available and the model is updated, index values of earlier years are not affected. Otherwise, if the historical values are revised again and again, it is impossible to track the indices. There are two reasons why the CBD model demonstrates such property. Firstly, the model's log-likelihood does not have the same parameters for two different points of time. Thus, extending the sample period does not change the maximum likelihood estimates previously calculated. Secondly, the model does not require any parameter constraint, which means that there is no need to revise previous parameter values.

The two parameters can readily be interpreted. The first index $k_t^{(1)}$ refers to the *level* of the logit mortality curve. A decrease in $k_t^{(1)}$ indicates an overall mortality improvement. The second index $k_t^{(2)}$ refers to the slope of the logit mortality curve. An increase in $k_t^{(2)}$ implies that mortality at younger ages improves faster than at older ages. As shown in Figure 1, the two indices can be used to express a logit mortality curve with any level and slope. Figure 2 illustrates the mortality curves in the original scale under four different pairs of index values. It can be seen that the two indices can jointly capture varying age patterns of the mortality curve.

Figure 1: Changes in the logit mortality curve when one or both of the two indices change

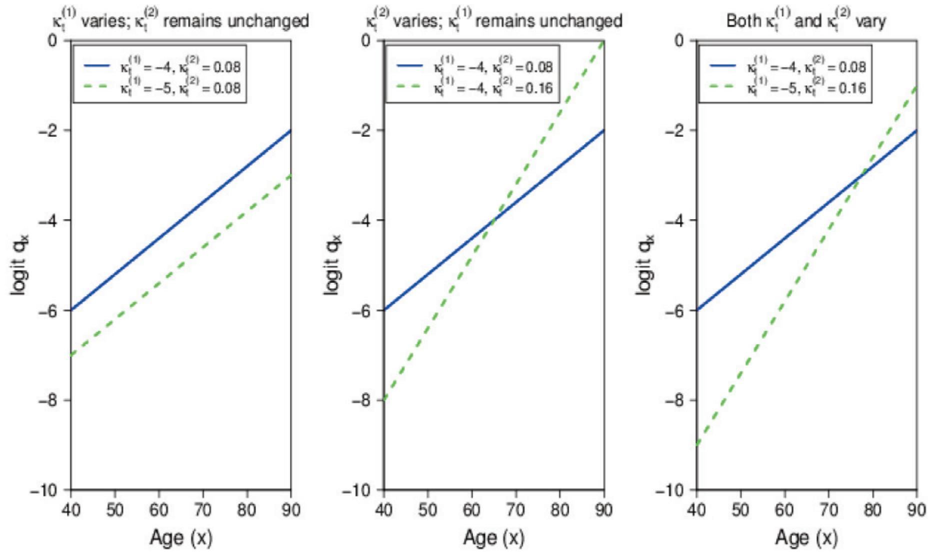
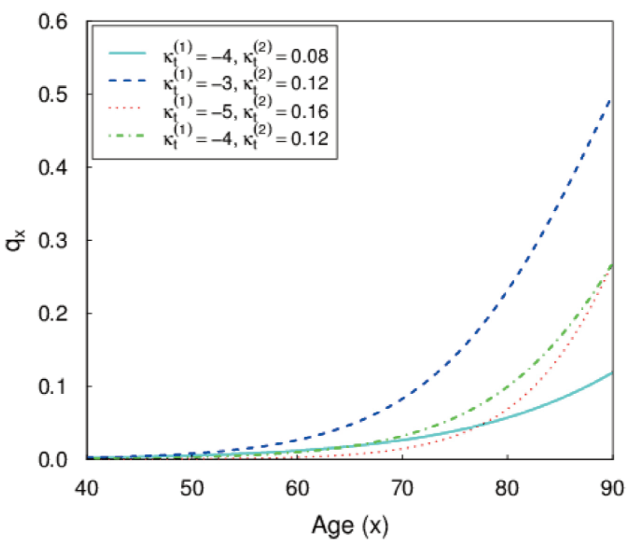


Figure 2: Mortality curves under different pairs of indices



For a pension plan, payouts are larger when overall mortality improvement is greater than expected (i.e. $k_t^{(1)}$ is lower than expected) and when mortality improves faster at older ages (i.e. $k_t^{(2)}$ is lower than expected). For a term-life insurance (typically sold to young people) provider, payouts are larger when overall mortality improvement is less than expected (i.e. $k_t^{(1)}$ is higher than expected) and when mortality improves slower at younger ages (i.e. $k_t^{(2)}$ is lower than expected).

3. Multivariate time series models

In earlier studies, the CBD time-varying parameters are usually modelled with a bivariate random walk with drift. This simple model, however, does not take proper account of the serial- and cross-correlations. Instead, we propose to use the vector autoregressive integrated moving average (VARIMA) models [4]. To start with, the vector autoregressive moving average VARMA(p , q) model for a bivariate time series $Z_t = (k_t^{(1)}, k_t^{(2)})'$ is expressed as

$$Z_t = C_0 + \sum_{i=1}^{p} \Phi_i \, Z_{t-i} + \sum_{j=1}^{q} \Theta_j \, \varepsilon_{t-j} + \varepsilon_t ,$$

where C_0 is a (2 × 1) intercept vector, and Φ_i's and Θ_j's are (2 × 2) autoregressive and moving average coefficient matrices respectively. The residual (2 × 1) vectors εt are independently and identically distributed as bivariate normal with mean zero and covariance matrix Σ. The order (*p, q*) can be obtained by the Tiao and Box approach [3]. The VARMA model is appropriate if Z_t is weakly stationary, i.e. the mean vector and covariance matrix of Z_t are constant across time. Otherwise, we may convert it into a stationary one via differencing, i.e. modelling $\Delta Z_t = Z_t - Z_{t-1}$ rather than Z_t. If ΔZ_t is still non-stationary, we can further use $\Delta^2 Z_t = \Delta Z_t - \Delta Z_{t-1}$. Suppose *d* is the times of differencing to make it stationary. The resulting VARIMA(*p, d, q*) model is expressed as

$$\Delta^d Z_t = C_0 + \sum_{i=1}^{p} \Phi_i \, \Delta^d Z_{t-i} + \sum_{j=1}^{q} \Theta_j \, \varepsilon_{t-j} + \varepsilon_t .$$

To avoid model mis-specification, diagnostic testing of the residuals is required. If the final model provides an adequate fit, the residuals are free from significant serial- and cross-correlations.

4. Index-linked derivatives

We propose a standardised derivative called K-forward, which exchanges a fixed amount for a random amount proportional to a CBD mortality index. For the fixed receiver, the payoff is

$$X \times (\tilde{\kappa}_{t^*}^{(i)} - \kappa_{t^*}^{(i)}), \; i = 1, 2,$$

where *X* is the notional amount, *t** is the maturity date, and $\tilde{\kappa}_{t^*}^{(i)}$ the forward value. For example, an annuity provider or a pension plan may enter into a K-forward as a fixed receiver – payouts from K-forwards can offset the adverse situation when both indices turn out to be lower than expected. The calibration of K-forwards can be performed by adopting modified versions of duration and convexity measures.

Acknowledgements

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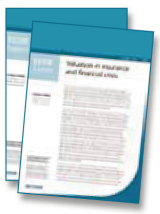

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